ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

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SEMINAR OF NUMERICAL ANALYSIS

- **WEDNESDAY 29 MARCH 2017 - ROOM ME D0 1418 - 16h15**

Prof. Alberto Valli (Universita' degli Studi di Trento, Italy) will present a seminar entitled:

« Eigenvalues of the curl operator: variational formulation and numerical approximation »

Abstract:
In electromagnetism, for linear isotropic media the relation between the magnetic induction ***B*** and the magnetic field ***H*** is given by ***B = uH***, the scalar function *u* being the magnetic permeability. If displacement currents are neglected, as in the case of eddy current problems, the current density ***J*** is given by ***J*** = **curl*H***. In this situation a magnetic field satisfying **curl*H = nH***, with *n* a scalar function, produces a vanishing magnetic force ***J x B***, and it is called a force-free field.
In fluid dynamics, a divergence-free field *u* satisfying **curl *u = nu***, with *n*
a scalar function, is a steady solution of the Euler equations for incompressible
inviscid ows (with pressure given by $p = -\frac{|u|^2}{2}$ and it is called a Beltrami field.

Eigenfunctions of the curl operator are therefore force-free fields and Beltrami fields, and are of relevant physical interest. In particular, in plasma physics a magnetic field ***H*** which minimizes the magnetic energy with fixed helicity has to satisfy the equation **curl*H = λH*** for some constant λ, thus it is an eigenfunction of the curl operator.

In this talk we are concerned with two topics: the formulation and analysis of the eigenvalue problem for the curl operator in a multiply-connected domain, and its numerical approximation by means of finite elements. Following the results in [1] and extending the previous ones in [2], [3], [4], we prove that the curl operator is self-adjoint on suitable Hilbert spaces, each of them being contained in the space of vector fields *v* for which curl *v* x n = 0 on the boundary.
An important point is to note that additional conditions must be imposed when the physical domain is not topologically trivial: a viable choice is the vanishing of the line integrals of *v* on suitable homological cycles lying on the boundary.
We devise and analyze a saddle-point variational formulation, and propose a finite element numerical scheme, generalizing the results in [5], [6]. We prove that eigenvalues and eigenfunctions are efficiently approximated, and present some numerical results for testing the performance of the method.

Lausanne, 2 March 2017/AB/ms

The seminars are announced at http://mathicse.epfl.ch/seminars