When performing large-scale physical simulations, one often encounters linear systems that are so large that they must be subdivided and solved in parallel using many processors. In optimized Schwarz methods, this is done by dividing the computational domain into many subdomains, solving the smaller subdomain problems in parallel, and iterating until one obtains a global solution that is consistent across subdomain boundaries. Fast convergence can be obtained if Robin conditions are used along subdomain boundaries, provided that the Robin parameters $p$ are chosen correctly. For the Poisson equation, it is well known that two-subdomain problems with no overlap, the optimal choice is $p = O(h^{-1/2})$ (where $h$ is the mesh size), with the resulting method having a convergence factor of $\rho = 1 - O(h^{1/2})$.

However, when cross points are present, i.e., when several subdomains meet at a single point, this choice leads to a divergent method. In this work, we show for a model problem that convergence can only occur if $p = O(1/h)$ at the cross point; thus, a different scaling of the Robin parameter is needed to ensure convergence. In addition, this choice of $p$ allows us to recover the $1 - O(h^{1/2})$ convergence factor in the resulting method.